NATIONAL BUSINESS AND TECHNICAL EXAMINATIONS BOARD
NBC/NTC EXAMINATION MATHEMATICS

1. Use the logarithm tables to evaluate

$$\sqrt[4]{\frac{0.784^3 \times 23.67}{3.479}}$$

<u>Solution</u>

No	Log
$(0.784)^3$	$\bar{1}.8943 \times 3 = \bar{1}.6827$
23.67	(+) 1.3742
	1.0571
3.479	0.5414 (-)
1.346	0.5157 ÷ 4 = 0.1289

Antilog of 0.1289 = 1.346

2(a) Find the product of 324_6 and 15_6

(b) If log a +5 log a - 6log a = log 8. What is a?

<u>Solution</u>

$324_6 = 3 \times 6^2 + 2 \times 6^1 + 4 \times 6^o$

$= 3 \times 36 + 12 + 4 = 124_{10}$

$15_6 = 1 \times 6^1 + 5 \times 6^o = 6 + 5 = 11_{10}$

$\therefore \quad 124_{10} \times 11_{10} = 1364_{10}$

Then, next

6	1364	
6	227	2
6	37	5
6	6	1
6	1	0
		1

$\therefore 324_6 \times 15_6 = 10152_6$

3(a) Make T the subject of the expression:

$$N = \sqrt{\left(\frac{S}{T} - \frac{P}{Q}\right)}$$

(b) If S is directly proportional to T and T = 120, when S = 30;
Find the value of T when S = 136

Solution

(a) $N = \sqrt{\left(\frac{S}{T} - \frac{P}{Q}\right)}$

Clearing the root sign

$N^2 = \frac{S}{T} - \frac{P}{Q}$

$\frac{S}{T} = N^2 - \frac{P}{Q}$

$\frac{S}{T} = \frac{QN^2 + P}{Q}$

Multiply both sides by 1/S and reciprocal the expression or cross multiply, we have

$T = \frac{SQ}{QN^2 + P}$

(b) $S \propto T$

S = KT

30 =K x 120

$\frac{30}{120} = K$

$K = \frac{1}{4}$

$S = \frac{T}{4}$

∴ S = ¼ T

If T = 120

When S = 136 then 136 = ¼ T

∴ T = 544

4(a) Evaluate $10.5^2 – 1.5^2$, without the use of Mathematical tables

(b) Expand $\left(a+2\sqrt{3}\right)\left(a-3\sqrt{2}\right)$

Solution

4(a) $10.5^2 – 1.5^2$ is a difference of two square values

$\Rightarrow$ (10.5 +1.5) (10.5 – 1.5) = (12.0) (9.0)

= 108

(a) Expanding $\left(a+2\sqrt{3}\right)\left(a-3\sqrt{2}\right)$, we have

$a^2 - 3a\sqrt{2} + 2a\sqrt{3} - 6\sqrt{6}$

5(a) Calculate the area of the major sector of a circle which subtends an angle of 130^o at the centre and having radius 14cm. (Take π to be 3.14)

(b) Rationalize $\frac{2}{4+3\sqrt{2}}$

Solution

5. (a) $\frac{\theta}{360^0} \times \pi r^2$

$= \frac{130}{360^0} \times 3.14 \times 14^2$

$= 222.24cm^2$

b) $\frac{2}{4+3\sqrt{2}} = \frac{2(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$

$= \frac{8-6\sqrt{2}}{16-12\sqrt{2}+12\sqrt{2}-9\sqrt{4}}$

$= \frac{8-6\sqrt{2}}{16-18}$

$= \frac{8-6\sqrt{2}}{-2} = \frac{2(4-3\sqrt{2})}{-2}$

$= 3\sqrt{2}-4$

6(a) Factorise completely $(x^2 + x)^2 - (2x + 2)^2$

(b) Express a in terms of x, b and y, if $\frac{a+x}{a-x} = \frac{y-b}{y+b}$

(c) Two places on the equator are 7900km apart measured along the equator. Find the difference in their longitudes. Take R = 6370km and π = 3.14

Solution

(a) $(x^2 + x)^2 - (2x + 2)^2$

$= (x^2 + x)(x^2 + x) - (2x + 2)(2x + 2)$

$= (x^4 + x^3 + x^3 + x^2) - (4x^2 + 4x + 4x + 4)$

$= (x^4 + 2x^3 + x^2 - (4x^2 + 8x + 4)$

$= x^4 + 2x^3 - 3x^2 - 8x - 4$

$= (x^2 + 3x + 2)(x^2 - x - 2)$

$= (x + 2)(x - 2)(x + 1)(x + 1)$

$=(x + 2)(x - 2)(x + 1)^2$

(b) If $\frac{a+x}{a-x} = \frac{y-b}{y+b}$

Cross multiplying, $(a + x)(y + b) = (a - x)(y - b)$

$ay + ab + xy + xb = ay - ab - xy + xb$

By collecting like term and solving for a, we have ab = -xy

$$\therefore a = \frac{-xy}{b}$$

(c) $\frac{\theta}{360^0} \times 2 \times 3.14 \times 6370 = 7900$

$$\therefore \theta = \frac{7900 \text{ x } 360^o}{2 \text{ x } 3.14 \text{ x } 6370}$$

Simplifying, we obtain

$= 71.09^o \approx 71.1^0$

7(a) Find the sum of the first three terms of the G.P whose third term is 27 and whose 6^{th} term is 8.

(b) A cone is formed by folding a major sector of a circle having an angle 220^o at the centre. Calculate the circumference of the base of the cone if the diameter of the circle is 14cm, correct to 1 decimal place.

Solution

(a) Using ar^{n-1}

The third term is $ar^2 = 27$______(i)

And the 6^{th} term is $ar^s = 8$_______(ii)

Solving, we obtain

r = 2/3

Solving for a in equation, we have

$$a\left(\frac{2}{3}\right)^2 = 27$$

$$\therefore a = \frac{243}{4} = 60\frac{3}{4} \text{ or } 60.75$$

Sum of the terms = $60\frac{3}{4} + \frac{243}{4}\left(\frac{2}{3}\right) + 27$

or $60\frac{3}{4} + \frac{243}{4}\left(\frac{2}{3}\right) + \frac{243}{4}\left(\frac{2}{3}\right)^2$

Evaluating, we have

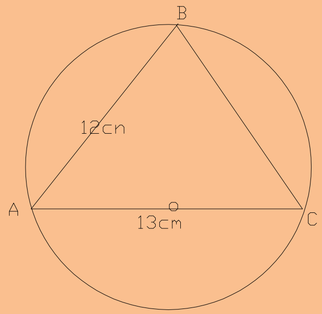
= 1284

(b) radius of circle = 7cm

$\therefore$ Circumference of the base of cone = $\frac{220^0}{360^0} \times \frac{22}{7} \times \frac{7}{1}$

= 26.9cm (correct to 1 decimal place)

8(a) In the diagram ABCD is a circle centre 0 with diameter 13cm. ABC is a triangle inscribed in the circle.



Find, correct to 3 significant figures, the
(i) area of the triangle ABC
(ii) total area of the shaded portion and
(iii) perimeter of the shaded area ACD

8(b) Simplify without using Mathematical tables the sum of the first 20 terms of the series 3 + 6 + 9 + 12 + …

Solution

8(a) (i) $/BC/ = \sqrt{13^2 - 12^2} = 5cm$

$\therefore$ Area of triangle ABC = (½ x 12 x 5) cm

= $30.0cm^2$

(ii) Area of circle: r = 6.5cm

$= \pi r^2 = \frac{22}{7} \times (6.5cm)^2$

$= 132.7495cm^2 \approx 132.75cm^2$

Area of the shaded portion = $(132.75 - 30)\ cm^2$

$= 102.75cm^2 \approx 103cm^2$ (to 3 sig. fig)

(iii) Length of arc ADC = $\frac{1}{2} \times 2\pi \times \frac{13}{2}$ cm

= 20.42cm

$\therefore$ the perimeter = 20.42cm + 13cm

= 33.42cm

$\approx$ 33.4 cm (3sig. fig.)

(c) Using $S_n = \frac{n}{2}[a + (n-1)d]$

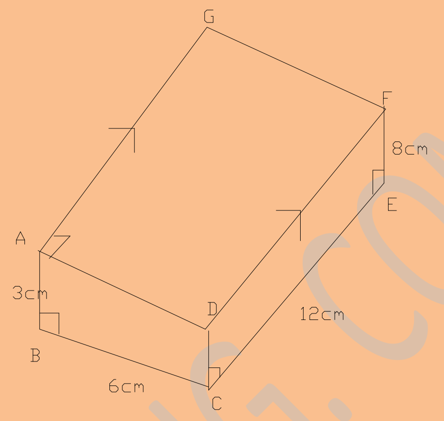
$$S_{20} = \frac{20}{2}[6 + (20-1)3]$$

$\therefore$ $S_{20} = 630$

9(a) With a pair of compasses and ruler only, construct a triangle PQR in which <RPO = 120°, <PQR = 45° and /PQ/ = 6cm

(i) Find a point M on RQ such that PM is perpendicular to RQ.

(ii) Measure /PM/

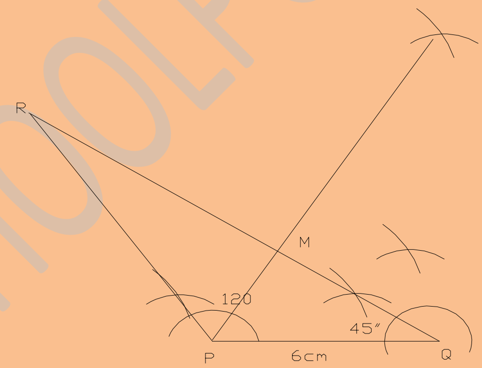


(b) The figure given above is a solid with CEFD as the cross section. Calculate the:

(i) area of CEFD, and

(ii) volume of the solid.

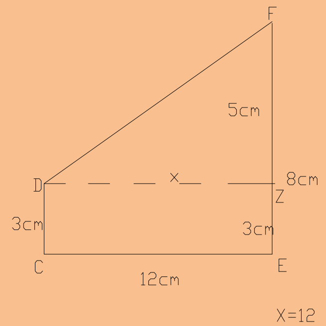
<u>Solution</u>



9(a)

(ii) /PM/ = 4.2cm ($\pm$ 0.1cm)

(b) Area of CEFD



9(b) (i) Area of CDZE = l x b = (3 x 12)cm = $36cm^2$

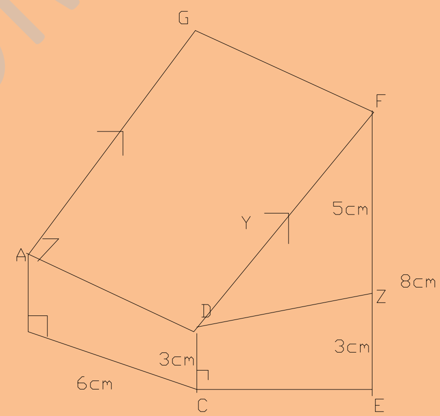
Area of DFZ = ½ b x h

= ½ x 5 x 12 = $30cm^2$

∴ Area of CEFD = $36cm^2 + 30cm^2$

= $66cm^2$

(ii)



$y = \sqrt{144 + 25} = 13$

Volume of cuboid ABCDZE = 3cm x 12cm x 6cm

= $216cm^3$

Volume of ADZFG = volume of ½ prism

½ volume of prism = ½ l x b x h (h=height) = ½ x (6 x 5 x 12)cm

= $180cm^3$

$\therefore$ the volume of the solid = $216cm^3 + 180cm^3$
= $396cm^3$

10(a) *If* $\xi = \{1,2,3,\ldots,10\}$ and A = {4,6,8,10}, B = {1,4,5,11}, C = {4,5,11,12}, find $C^1 \cup (A \cap B)$

(b) Solve graphically, the simultaneous equations:
$y = x^2 - 7x + 10$ and $y = x + 3$ using the interval $0 \leq x \leq 8$ and a scale of 2cm to 1 unit on the x - axis and 1cm to 2 units on the y - axis.

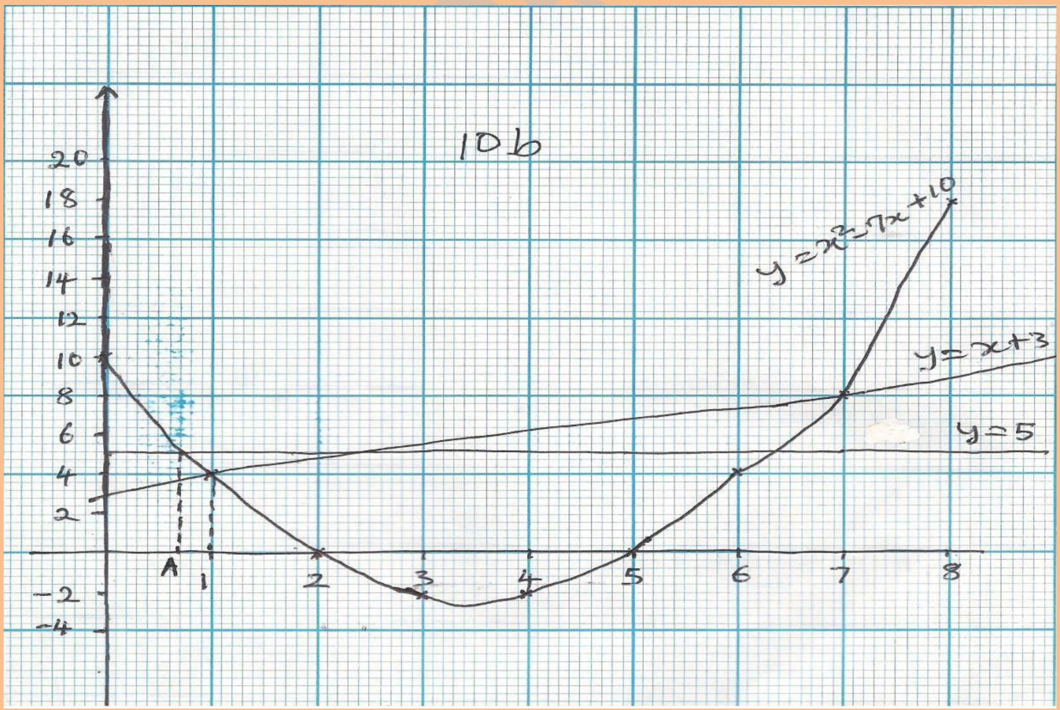
(c) Use your graphs in (a) to find the roots of :
(i) $x^2 - 7x + 10 = 0$
(ii) $x^2 - 7x + 5 = 0$

Solution

10(a) $C^1 = \{1, 2, 3, 6, 7, 8, 9, 10\}$
$A \cap B = \{4\}$
$C^1 \cup (A \cap B) = \{1, 2, 3, 4, 6, 7, 8, 9, 10\}$

(b) Table of values: $y = x^2 - 7x + 10$

x	0	1	2	3	4	5	6	7	8
y	10	4	0	-2	-2	0	4	10	18



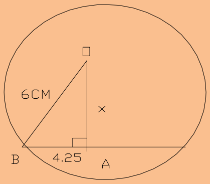
(c) (i) x = 2, x=5 $\pm$(0.1cm)
(ii) the roots are given by x = 0.7, x = 6.2 $\pm$ (0.1cm)

11(a) In a circle of radius 6cm, calculate the distance from the centre to a chord which is 8.5cm long.

(b) A sum of ₦154,000 was to be shared among three children, Hassan, Victor and Garba such that Hassan receives 2/3 of Victor's share; while Victor receives ½ of Garba's share. How much is received by Victor?

Solution

Let the distance be x



$$x = \sqrt{6^2 - 4.25^2}$$
$$= \sqrt{17.9375}$$
$$= 4.24\text{cm}$$

11(b) Suppose Garba receives ₦ x (or equivalent). The equation will be
x + ½x + 1/3 x = 154,000
Simplifying, we have
$\frac{11}{6}x = 154,000$
11x = 154,000 x 6
$\therefore$ x = 84,000
$\therefore$ Victor receives ½ x ₦84,000
= ₦42,000.00

12(a) The distribution of the daily wages in ₦100 of some workers on a farm is as given below.

Wages (x)	2	3	4	5	6	8	10
No. of workers (f)	2	4	10	11	15	10	3

(a) How many workers are on the farm?
Calculate the:
(i) mean wage
(ii) median wage and

(iii) modal wage

(b) The monthly profit of a transport business was shared between two partners, a husband and wife in the ratio 7:5. If the wife received ₦15,000 less than the husband, find out how much the husband received.

Solution

(a) Total number of workers in farm = 2 + 4 + 10 + 11 + 15 + 10 + 3
= 55

(b) (i) mean wage

$\sum fx = (2 \times 2) + (3 \times 4) + (4 \times 10) + (5 \times 11) + (6 \times 15) + (8 \times 10) + (10 \times 3)$
= 31100

$\therefore$ mean = $\frac{\sum fx}{n} = \frac{31100}{55}$ = ₦565.45

(ii) Median wage:
In the middle position, we have ₦600.00

(iii) Modal wage: The most frequent wage was ₦600.00

(b) Let the husband receive ₦x. Then we have the equation $\frac{x}{x-15000} = \frac{7}{5}$

$\Rightarrow 5x = 7x - 105000$
$\therefore x = 52,500$
Then, the husband received ₦52,500.00

13(a) A trader bought 98 units of an article at ₦180 each. He sold 42 of them at a profit of 20%, 35 at a loss of 4% and remainder at a profit of 15%. Find the overall
(i) selling price to nearest kobo, and
(ii) percentage gain or loss to 2 decimal places.

(b) A simple interest on a sum of money invested at 4% for 4 years was ₦4,040. How much was invested?

Solution

(a) (i) Selling price of 42 articles = $\frac{120 \times 7560}{100}$ = 9072 $\Rightarrow$ ₦9072

Selling price of 35 articles = $\frac{96 \times 6300}{100}$ = ₦6048

Selling price of remaining 21 articles = $\frac{115 \times N3780}{100}$ = ₦4347

$\therefore$ Overall selling price = ₦9072 + ₦6048 + ₦4347 = ₦19,467.00

(ii) The overall cost price of the articles = ₦7560 + 6300 + 3780
= ₦17640.00

(or ₦180 x 98 = 17640)

Overall gain = selling price – cost price

= ₦19467 – ₦17640 = ₦1827

$\therefore$ percentage gain = $\frac{1827}{17640} \times \frac{100\%}{1}$

= 10.36%

(b) S.I. = $\frac{P \times T \times R}{100}$

$\therefore P = \frac{S.I \times 100}{T \times R} = \frac{100 \times 4040}{4 \times 4}$

$\therefore$ P = 25,250.00

14(a) A married man with 5 children is on an annual salary of ₦75,000. The man is given tax relief as follows:

Personal Allowance of ₦9,000

Children Allowance of ₦1,500 per child for a maximum of 4 children.

Dependent Relative Allowance of 1/10th of his salary.

Life Insurance Allowance of ₦5,000.

If tax is paid at 10k in ₦ on the 1st ₦20,000 and 15k in ₦ on the remaining, calculate the amount of tax he pays.

(c) A trader allows a retailer 20% trade discount and 5% for cash payment. What will be the marked price of an article for which a customer pays ₦4,750?

Solution

14(a) Tax free allowance

Personal allowance	=	₦9,000
4 children allowance @ N150 per child	=	₦6,000
1/10 of salary for dependent relative allowance	=	₦7,500
Life Insurance allowance	=	₦5,000
		₦27, 500

Taxable income = ₦75,000 – ₦27,500

= ₦47,500

Tax on first ₦20,000 = $\frac{20,000 \times 10}{100}$

= ₦2000

Remaining: ₦47, 500 - ₦20,000 = ₦27, 500

Tax on remaining amount = $\frac{N27500 \times 15}{100}$ = ₦ 4125

$\therefore$ total tax paid = ₦2,000 + ₦4125

= ₦6125.00

(b) Let the marked price be ₦x.

$$\frac{95}{100} \times \frac{80x}{100} = 4750$$

$$95 \times 80x = 4750 \times 100 \times 100$$

$$\therefore x = \frac{4750 \times 100 \times 100}{95 \times 80}$$

Simplifying, we have x = 6250

$\therefore$ the marked price = ₦6250.00

15(a) A and S declares a cash dividend of ₦200,000 in a certain year as follows: The 1000 shares of preferred stock are to receive 6% of the ₦250 per value. While the 5,000 shares of ordinary stock are to receive the remainder. Calculate the annual dividend per share for each type of stock.

(b) Obi and Audu own a shop. The ratio of Obi's share to Audu's share is 13:7. Later Audu sells 1/5 of his shares to Obi for ₦6,300. Find the value of the shop.

Solution

(a) For the first preferred stock at 6%

$$\Rightarrow \frac{6}{100} \times N250 = N15$$

Dividend on the preference shares = ₦15 x 1000 = ₦15,000

Dividend on the ordinary shares: = ₦200,000 – ₦15,000 = ₦185,000

$\therefore$ Dividend per share would be $\frac{₦85,000}{5,000}$

= ₦37.00

(b) Let the value of the shop be ₦x

let Audu's share be ₦$\frac{7x}{20}$

$$\therefore \frac{1}{5} \times \frac{7x}{20} = 6300$$

Solving, we obtain x = 90,000

$\therefore$ the value of the shop is ₦90,000.00